BOOK REVIEWS.

Analytic Mechanics. By JOHN ANTHONY MILLER, Ph.D., Professor of Mathematics, Swarthmore College, and SCOTT BARRETT LILLY, C.E., Assistant Professor of Engineering, Swarthmore College. D. C. Heath and Company, 1915.

The motive of the authors of this book is described in the preface in the following words: "We have attempted to write a rigorous, teachable introduction to the study of mechanics. We believe that certain fundamental principles of mechanics, used in common by students in the various branches of engineering, in theoretical physics, and in celestial mechanics are essential to the satisfactory progress of a student in any of these fields. To this end, we have chosen as our subject matter only such fundamental theorems." The result is a book of 292 pages, consisting of an introduction and sixteen chapters treating of the following topics: Composition and resolution of forces acting on a particle; statics of a particle; forces acting on a rigid body; vectors; statics of a rigid body; center of gravity; friction; flexible cords; kinetics of a particle; motion of a particle in a plane curve; work and energy; dynamics of a rigid body; kinetic friction. The difficult problem of choice of material has been solved with sufficient success to produce a book without important omissions and of usable dimensions. A number of excellent treatises on mechanics have been published in the English language during the past generation, but the best of these have been unadapted for use as introductory textbooks because of their unmanageable dimensions. Of the numerous attempts which have been made to meet the demand for a satisfactory introductory course, the book of Miller and Lilly is among the most successful in the important matter of selection of material.

The authors have also attained a considerable degree of success in their attempt "always to be rigorous" while producing "a book that is distinctly teachable." Teachable it undoubtedly is; explanations are stated in simple, direct language; too much is not presupposed in the way of mathematical knowledge and acumen on the part of the student. The exercises for the student are for the most part sufficiently simple and designed to illustrate the principles of mechanics rather than the elegance of a mathematical method. It is mainly in the explanation of principles, rather than in the application to particular cases, that there occur what the reviewer regards as lapses of rigor.

The treatment shows the sympathy for practical applications which would be expected from the fact that one of the authors is a teacher of engineering. Engineering applications are however brought in incidentally rather than by formal separation in the treatment; thus the determination of stresses in jointed frames is explained in the chapters on statics of a particle and statics of a rigid body.

The laws of motion are stated in the language of Newton (the English rendering being nearly identical with that of Thomson and Tait), and very little is said by way of explaining the meaning of the laws. The equation of motion

for a particle is habitually employed in the form $F = (W/g)a$; but this is deduced from the equation $F = kma$ which is given as the algebraic expression of the second law and is thus implicitly taken as the "fundamental" equation. The explanation of basic principles may perhaps fairly be characterized as traditional; that is, it is in the main the method which has predominated in English and American books since the publication of Thomson and Tait's *Natural Philosophy*. To a considerable extent the lapses of rigor referred to above are characteristic of this traditional method. It seems worth while to refer in some detail to certain of the defects which, according to the view of the present writer, have been perpetuated in this traditional treatment. These defects are associated with the following subjects: (1) The concept of force, (2) the composition of velocities and accelerations, (3) the theory of work and energy.

1. The vagueness of the definition of force is perhaps the most serious defect in the traditional treatment. A proper explanation should wholly remove this vagueness. The practical concept of force involves the following points: (*a*) A force is a push or a pull exerted upon a body or portion of matter; (*b*) every force acting on one body is exerted by some other body (*i. e.*, there is some body which does the pushing or pulling); (*c*) when a body A is pulling or pushing a body B, the body B is at the same time pulling or pushing A equally in the opposite direction and along the same line; (*d*) a force tends to change the velocity of the body upon which it acts. The traditional definition and explanation usually give no explicit statement of either (*b*) or (*c*)—often scarcely a hint of these important facts—and in many cases there is no appeal even to the notion of push or pull in explaining force.[1]

The book under review shares the traditional vagueness in the explanation of force. At the beginning of Chapter I we read: "Force is that which changes, or tends to change, the motion of matter. It is either a push or a pull." There is no hint here or in the context that two bodies or portions of matter are concerned, nor any intimation that the real meaning of the law of action and reaction is the above statement (*c*).[2] Moreover the explanation is marred by statements which are not merely vague but incorrect: "That which moves matter is *force*" (p. 4); "In a general way we have known that if a body is moving it is acted upon by some force" (p. 6).

The vagueness which results from a failure to keep definitely in mind the points above designated as (*b*) and (*c*) appears in the treatment of internal and external forces in the dynamics of a rigid body (Art. 249). There is no clear

[1] An examination of six books taken at random among textbooks on mechanics published during the past ten years failed to find in any one the use of the words push or pull to explain force; and it is fair to say that the majority of books either give no hint at all that two bodies are concerned in the action of every force or refer to it merely incidentally. Undoubtedly many students have completed a course in mechanics without having their attention called either to this fact or to the real meaning of the law of action and reaction.

[2] Some textbook writers appear to take it for granted that the explicit statement (*c*) is unnecessary because sufficiently implied by the statement "to every action there is an equal and contrary reaction." Nothing is more certain, however, than that able writers have repeatedly been led into mistakes by the failure to recognize that (*c*) is the real meaning of the third law.

statement of the essential distinction between these two classes of forces (that an internal force is exerted upon some part of the body whose motion is being studied *by another part of the same body,* while an external force is exerted *by some other body*). When this is clearly stated, and when the law of action and reaction is understood to have the meaning (*c*), it is at once seen that the internal forces cancel out of the equations obtained by summing the equations of motion written for all individual particles. This cancellation depends merely upon the fact that, for any pair of forces constituting action and reaction, the vector sum is zero and the sum of the moments about any axis is zero; a serious confusion of language, if not of thought, is involved in the statement that "the internal forces are in equilibrium among themselves."[1]

2. The vagueness of the traditional treatment of the composition of velocities is well illustrated by the words "if a particle is subject to two simultaneous velocities" (p. 158 of the book under review). It is true that such language is very common and that there is a stock explanation of its meaning. This explanation refers to a particle moving with a certain velocity with respect to a body which is itself in motion. Thus if a particle is supposed to "move along a rod with uniform speed" while the rod is "carried parallel to itself" with uniform velocity, it is easily seen that the actual velocity of the particle is the vector sum of the velocity of the rod and the velocity of the particle with respect to the rod. There is no vagueness here; but the explanation throws no light whatever upon the meaning of simultaneous velocities *with respect to a single base or body of reference.* Yet this is precisely what needs explanation, since the language commonly employed implies that a body may have at the same time two different velocities *with respect to the same base.* The fact is that, so long as motion is referred to one definite base, a particle has at any instant one definite velocity. This actual velocity, being a directed or vector quantity,[2] may *for the purposes of mathematical analysis or computation* be resolved into components by the rules of vector addition; but this does not at all warrant the statement that a particle has (or is "subject to") two or more different velocities at the same time.[3]

What has just been said of velocity applies also to acceleration, so far as the meaning of vector composition and resolution is concerned. An acceleration is

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[1] The statement that an action and reaction constitute a pair of counterbalancing forces is one of the most unfortunate errors which are perpetuated by the carelessness of able writers. Action and reaction never counterbalance each other; they act on different portions of matter.

[2] The fact that "velocity is a vector" is not a proposition requiring proof (as might be inferred from the language used on p. 159); the very conception of the instantaneous motion of a particle makes its velocity a directed magnitude.

[3] The logical point involved may be illustrated by an analogy. The position of a particle may be specified by a vector OA, drawn from a fixed origin O to the instantaneous position A, and this position-vector may be expressed as the sum of two vectors OB, OC so taken that $OBAC$ is a parallelogram. If we were to interpret this as meaning that the particle at A may be regarded as occupying simultaneously the positions B and C, we should be making the same kind of statement that is made when it is said that a particle has at the same instant two different velocities.

It may be remarked also that the use of the words "subject to a velocity" seems to indicate a confusion of thought, since they imply that the continuance of a velocity must be due to some external cause—a notion explicitly negatived by the first law of motion.

a vector quantity and may, for the purposes of analysis or computation, be resolved into components; but this does not warrant the statement that a particle has at the same time two different accelerations with respect to one given base. *Apart from the idea of force* the resolution of the actual acceleration into vector components is an arbitrary geometric process. When we come to interpret the second law of motion as applied to a particle acted upon by more than one force, there is of course a special reason for treating the acceleration as made up of vector components, each associated with a particular force; the explanation of this fact is however not a proof that " acceleration is a vector " but is an explanation of the law of composition of forces. The idea that acceleration is a directed or vector quantity is purely kinematical—quite independent of any law involving force; in fact the full understanding of Newton's second law must presuppose the notion of acceleration defined as the rate of change of the velocity-vector.

3. In the traditional treatment there is a logical confusion associated with the definitions of work and energy. Energy is defined as *the ability of a body to do work,* while work is defined as being done not *by a body* but *by a force.*

In the book under review the chapter on Work and Energy begins with an admirably simple and clear elementary treatment of work done by a force. Kinetic energy is then defined by an algebraic formula, and a proof is given of the theorem that " the change in the kinetic energy of a particle equals the work done by the resultant force acting on it." Then follows a brief discussion of the condition for the existence of a force function U, leading to the definition of potential energy as " the negative of the function U." Thus far there is no lack of logical rigor. There follows however an Article headed " The Energy of a Particle is its Ability to do Work " in which we read: " It seems reasonable from the Third Law of Motion that the equations[1] . . . may be read the other way, viz., that in changing from the velocity v to the velocity v_0 the particle will do work on another body equal to w. This comes to saying that a particle will do as much work in giving up its velocity as has been done on the particle in order that it might acquire it." But what is meant by a particle " doing work upon a body "? This question is nowhere answered. The passage quoted is in fact fairly representative of the logical vagueness of the traditional treatment. To remove this vagueness it is necessary to adopt a clear *definition* of " work done by a body," from which it may be *proved* that the " ability of a particle to do work " (*i. e.*, the quantity of work it will do in coming to rest) is equal to $\frac{1}{2}mv^2$. With reference to the above quotation it is also to be said that the citation of the third law of motion is quite irrelevant and encourages confusion as to the meaning of that law.

The space occupied by the foregoing criticisms is out of proportion to their applicability to the book under review; the criticisms are in fact directed rather at the traditional treatment than at the example of it presented by this book. The usefulness of a book as a text depends far more upon the treatment of concrete

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[1] The equations referred to express the fact that " the change of the kinetic energy of a particle equals the work done by the forces acting on it."

applications than upon the logical rigor with which fundamental principles are established. The book of Miller and Lilly seems to be decidedly usable as a class textbook, and is likely to find favor among teachers.

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PROBLEMS AND SOLUTIONS.

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PROBLEMS FOR SOLUTION.

ALGEBRA.

468. Proposed by H. C. FEEMSTER, York College, Nebraska.

In each of the following series find the nth term and sum:

(a) $$2 + 5 + 9 + 15 + 24 + \cdots$$
(b) $$1 + 6 + 10 + 20 + 35 + \cdots$$
(c) $$1 + 5 + 15 + 35 + 70 + \cdots$$

469. Proposed by T. H. GRONWALL, New York City.

Show that the equation

$$f(x) = 2ax^4 + (1 - b)x^3 + b(1 - b)x - 2ab = 0,$$

where $0 < b < 1$, $a > 0$ and $a^2 > b$ has only one positive root and that this root lies between the roots of $g(x) = x^2 - 2ax + b = 0$.

470. Proposed by ERNEST W. BROWN, Yale University.

There are n numbers each lying between $-\frac{1}{2}$ and $+\frac{1}{2}$, such that any value of each between these limits is equally probable. What is the probability that their sum will lie between $s - \frac{1}{2}$ and $s + \frac{1}{2}$, where s is an integral multiple of $\frac{1}{2}$?

GEOMETRY.

499. Proposed by NATHAN ALTSHILLER, University of Oklahoma.

Find the surfaces all the plane sections of which are circles.

500. Proposed by R. T. McGREGOR, Bangor, California.

$OABC$, $OA'B'C'$ are two straight lines such that AA', BB', CC' are parallel. AB', $A'C$ meet in P; $A'B$ and AC' meet in Q. Show by synthetic projective geometry that PQ is parallel to AA'. MILNE's *Projective Geometry*, Chap. I, Ex. 20.

501. Proposed by R. P. BAKER, University of Iowa.

Find the minimum amount of lumber one inch thick required to pack a gross of spheres three inches in diameter in a rectangular box.

502. Proposed by R. P. BAKER, University of Iowa.

A designer of machinery requires a curve having the following properties:

(1) A closed curve touching a given circle at two diametral points and enclosing it.
(2) The sum of the three radii from the center of this circle to the curve which make with each other angles of 120° is constant.